Modal Block-LU-Decomposition Technique for the Efficient CAD of Ridged Waveguide Filters

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Abstract— **A mode-matching/block-LU-decomposition (MMLU) technique is presented for the fast calculation of the modal S-parameters of and the fields in waveguide components composed of step discontinuities and homogeneous sections. The MMLU approach leads to a numerically efficient block-tridiagonal matrix structure. Arbitrary cross-sections are included by the combination with the fast and flexible 2D FE method. The efficiency of the method is demonstrated at the CAD of advanced broadband higher-order mode suppression ridged waveguide filters. Excellent agreement with measurements verifies the theory.**

I. Introduction

The MODE-MATCHING method has proved to be a reliable technique for the CAD of the comprehensive class of waveguide components which are composed of homogeneous rectangular and/or circular waveguide sections, cf. e. g. [1] -[4]. For the combination of the individual subsections, the numerically stable generalized scattering [1], [4], or admittance matrix combination techniques [2] - [3] are typically applied.

This paper presents a mode-matching/block-LU-decomposition (MMLU) technique for the fast calculation of the overall modal S-parameters and modal fields of waveguide components which are composed of step discontinuities of arbitrary cross-section (Fig. 1). The MMLU approach is based on the whole set of equations for the normalized voltages and currents directly and leads to a numerically efficient block-tridiagonal matrix structure. The technique allows the direct calculation of the fields inside the structure (e.g. for high-power check purposes), is faster than the modal S- or Y-matrix combination, and enables the possibility to describe a reduced equation system for optimization runs without significant loss in accuracy.

Arbitrary waveguide cross-sections are included by the proven and fast hybrid mode-matching/finite element (MM/FE) method [4]. In contrast to other methods reported recently, like the boundary integral equation method, all desired eigenvalues are calculated directly by the solution of a linear sparse matrix eigenvalue problem, and no inefficient search algorithm is required. The initial 2D FEM mesh can locally be refined and optionally be smoothed. This allows the efficient inclusion of all kind of realistic waveguide cross-sections, such as ridged circular waveguide dual-mode filter coupling elements with sharp edges [4], or ridged sections with rounded corners for filters for high-power applications. At advanced high performance ridged waveguide filter examples (Fig. 1) it is demonstrated that the MMLU technique enables the convenient CAD of waveguide components where a high number of optimization runs is required in order to meet the increased return loss and rejection standards of the modern space and communication industry.

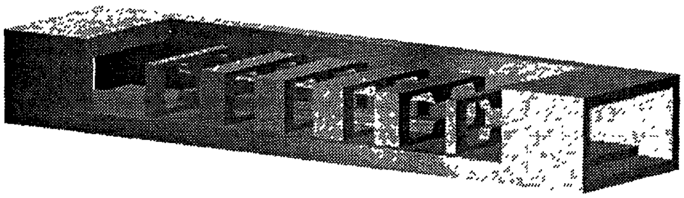

Fig. 1. Ridged waveguide five-cavity low-pass filter with rounded corners

II. Theory

The general waveguide structure under consideration (Fig. 2) is composed of N step discontinuities ($N \geq 2$), in which all waveguide sections may have arbitrary cross sections.

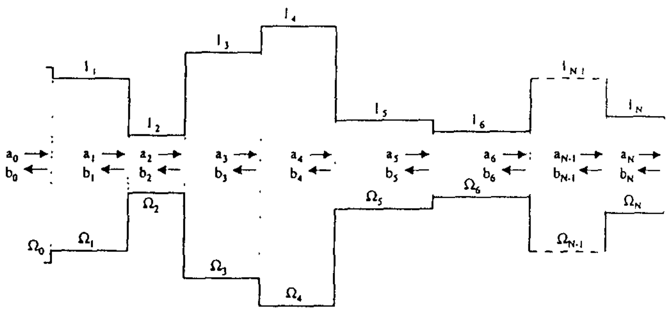

Fig 2 Step type waveguide structure

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Possibly overlapping structures are formulated by an intermediate section of zero length. The relations between the amplitude coefficients of forward and backward propagation waves arising from the application of the mode-matching technique [4] are

$$\begin{array}{l} \mathbf{E}_i^{-1} a_i + \mathbf{E}_i b_i = \mathbf{V}_{i-1\ i}^T (a_{i-1} + b_{i-1}), \\ \mathbf{V}_{i-1\ i} (\mathbf{E}_i^{-1} a_i - \mathbf{E}_i b_i) = a_{i-1} - b_{i-1}, \\ \mathbf{E}_i^{-1} a_i - \mathbf{E}_i b_i = \mathbf{V}_{i-1\ i}^T (a_{i-1} - b_{i-1}), \\ \mathbf{V}_{i-1\ i} (\mathbf{E}_i^{-1} a_i + \mathbf{E}_i b_i) = a_{i-1} + b_{i-1}, \end{array}$$

with

$$\begin{array}{l} \mathbf{V}_{i-1\ i} = \mathbf{diag}(\sqrt{Z_{i-1}}) \mathbf{K}_{i-1\ i}\, \mathbf{diag}(\sqrt{Y_i})\ \Omega_{i-1} \subset \Omega_i, \\ \mathbf{V}_{i-1\ i} = \mathbf{diag}(\sqrt{Y_{i-1}}) \mathbf{K}_{i-1\ i}\, \mathbf{diag}(\sqrt{Z_i})\ \Omega_{i-1} \supset \Omega_i, \\ \\ \mathbf{E}_i = \mathbf{diag}(\exp(-\gamma l_i))\ , i = 1\ \ldots\ N, \end{array} \tag{1}$$

where a_i and b_i are the amplitude coefficients of forward and backward propagating waves, γ is the propagation constant, l_i the length of section i and $\mathbf{K}_{i-1\ i}$ is the real and frequency independent coupling matrix between sections $i-1$ and i, and Z_i, Y_i are the field impedances and admittances, respectively, of section i.

Introducing modal voltages u_i and currents i_i in the usual way

$$\begin{array}{ll} u_i = a_i + b_i & \multirow{2}{*}{$\Omega_i \supset \Omega_{i+1}$} \\ i_i = a_i - b_i & \\ u_i = a_i - b_i & \multirow{2}{*}{$\Omega_i \subset \Omega_{i+1}$} \\ i_i = a_i + b_i & \end{array} \tag{2}$$

leads to the relations

$$\begin{array}{l} u_{i-1} = \mathbf{V}_{i-1\ i} (\mathbf{C}_i u_i + \mathbf{D}_i i_i) \\ \mathbf{V}_{i-1\ i}^T i_{i-1} = \mathbf{C}_i u_i + \mathbf{D}_i i_i \end{array} \tag{3}$$

with

$$\begin{array}{c} \mathbf{C}_i = \mathbf{diag}(\cosh(\gamma l_i)), \\ \mathbf{D}_i = \mathbf{diag}(\sinh(\gamma l_i)), \\ \text{if } \Omega_{i-1} \subset \Omega_i \subset \Omega_{i+1} \text{ or } \Omega_{i-1} \supset \Omega_i \supset \Omega_{i+1}\ , \\ \mathbf{C}_i = \mathbf{diag}(\sinh(\gamma l_i)), \\ \mathbf{D}_i = \mathbf{diag}(\cosh(\gamma l_i)), \\ \text{if } \Omega_{i-1} \subset \Omega_i \supset \Omega_{i+1} \text{ or } \Omega_{i-1} \supset \Omega_i \subset \Omega_{i+1}\ , \end{array} \tag{4}$$

and where Ω denote the corresponding cross-sections.

With eqns. (3), (4), and $\mathbf{C}_i^2 - \mathbf{D}_i^2 = \pm\mathbf{1}$, the final system of equations is given by:

$$\begin{bmatrix} \mathbf{P}_0 & \mp\mathbf{Q}_1 & & & & & \\ -\mathbf{Q}_1^T & \mathbf{P}_1 & \mp\mathbf{Q}_2 & & & & \\ & -\mathbf{Q}_2^T & \mathbf{P}_2 & \mp\mathbf{Q}_3 & & & \\ & & -\mathbf{Q}_3^T & \mathbf{P}_3 & & & \\ & & & & & \mp\mathbf{Q}_{N-2} & \\ & & & & -\mathbf{Q}_{N-2}^T & \mathbf{P}_{N-2} & \mp\mathbf{Q}_{N-1} \\ & & & & & -\mathbf{Q}_{N-1}^T & \mathbf{P}_{N-1} \end{bmatrix} \cdot \begin{pmatrix} u_0 \\ u_1 \\ u_2 \\ \cdot \\ \vdots \\ u_{N-2} \\ u_{N-1} \end{pmatrix} = \begin{pmatrix} 2a_0 \\ 0 \\ 0 \\ \vdots \\ \vdots \\ 0 \\ r_{N-1} \end{pmatrix}, \tag{5}$$

with

$$\begin{array}{l} \mathbf{P}_0 = \mathbf{1} + \mathbf{V}_{0\ 1} \mathbf{D}_1^{-1} \mathbf{C}_1 \mathbf{V}_{0\ 1}^T \\ \mathbf{P}_i = \mathbf{D}_i^{-1} \mathbf{C}_i + \mathbf{V}_{i\ i+1} \mathbf{D}_{i+1}^{-1} \mathbf{C}_{i+1} \mathbf{V}_{i\ i+1}^T \quad i = 1 \ldots N-1 \\ \mathbf{Q}_i = \mathbf{V}_{i-1\ i} \mathbf{D}_i^{-1}, \end{array} \tag{6}$$

and

$$\begin{array}{lll} \mathbf{C}_N = \mathbf{1}, & \mathbf{D}_N = \mathbf{1} & (a) \\ \mathbf{C}_N = \mathbf{diag}(\cosh(\gamma L_N)), & \mathbf{D}_N = \mathbf{diag}(\sinh(\gamma l_N)). & (b) \\ \mathbf{C}_N = \mathbf{diag}(\sinh(\gamma L_N)), & \mathbf{D}_N = \mathbf{diag}(\cosh(\gamma l_N)). & (c) \end{array}$$

$$\begin{array}{ll} r_{N-1} = \pm 2 \mathbf{V}_{N-1\ N} \mathbf{E}_N b_N & (a) \\ r_{N-1} = 0 & (b) \\ r_{N-1} = 0. & (c) \end{array}, \tag{7}$$

where the (a), (b), (c) designate

(a) port, upper / lower sign: $\Omega_{N-1} \subset / \supset \Omega_N$,
(b) short and $\Omega_{N-1} \subset \Omega_N$ or open and $\Omega_{N-1} \supset \Omega_N$.
(c) open and $\Omega_{N-1} \subset \Omega_N$ or short and $\Omega_{N-1} \supset \Omega_N$.

The upper sign of $\mathbf{Q}_i$ in equation (5) corresponds to equal step types at both ends of section i ($\Omega_{i-1} \subset \Omega_i \subset \Omega_{i+1}$ or $\Omega_{i-1} \supset \Omega_i \supset \Omega_{i+1}$), the lower sign to different step types. Since all submatrices $\mathbf{P}_i$ are symmetric, the resulting system of equations becomes symmetric too, if all subsequent equations are multiplied by -1, if different step types occur. This system of equations has a numerically efficient block-tridiagonal structure and has to be solved only for a few right hand sides (usually only for the fundamental mode incident at sections 0 or N). The set of equations (5) is solved by a $\mathbf{LTL}^T$ decomposition technique [5], with

$$\mathbf{L} = \begin{bmatrix} \mathbf{1} & & & & \\ \mathbf{L}_1 & \mathbf{1} & & & \\ & \mathbf{L}_2 & \mathbf{1} & & \\ & & & \cdot & \\ & & & \mathbf{L}_{N-1} & \mathbf{1} \end{bmatrix},$$

and

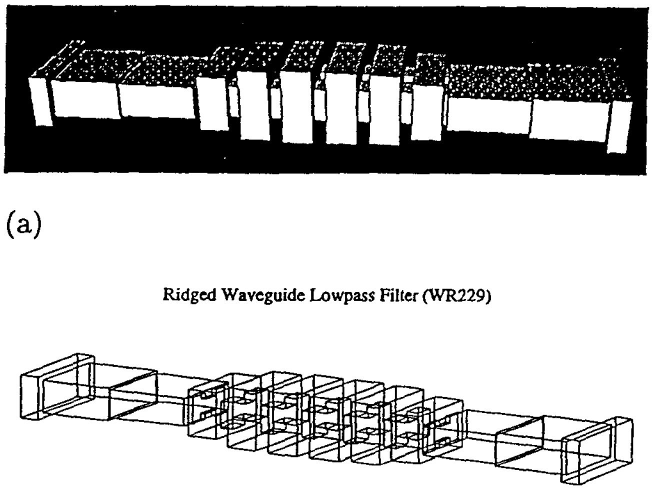
$$\mathbf{T} = \mathbf{diag}(\mathbf{T}_i).$$

The submatrices $\mathbf{L}_i$ and $\mathbf{T}_i$ are computed by the following scheme:

$$\begin{array}{ll} \mathbf{T}_0 = \mathbf{P}_0 & \\ \mathbf{L}_i = -\mathbf{Q}_i^T \mathbf{T}_{i-1}^{-1} & i = 1 \ldots N-1, \\ \mathbf{T}_i = \mathbf{P}_i - \mathbf{L}_i \mathbf{Q}_i & i = 1 \ldots N-1. \end{array} \tag{8}$$

From the solution vector of (5), and with equations (2), (3), the amplitude coefficients a_i, b_i can be computed. This yields, if required, also the modal fields at any location inside the homogeneous subsection i, which are then summed up e.g. for checking the maximum field strength.

Moreover, the solution of the system of equations (5) in its full-wave version and with the same set of modes in each subregion is approximately 2.6 times, or 1.6 times faster than the corresponding modal S- or Y-matrix combination, respectively. The third advantage is, that from the exponential character of the elements of the diagonal matrix $\mathbf{D}_i$ in equation (4) for higher order evanescent modes, it is evident, that the amplitude of such modes excited by the modes in section i-1 is negligibly small. This leads to the fact that only modes excited by the lower order modes in section i have to be considered.



(a)

(b)

Fig. 3. Ridged waveguide low-pass filter (WR-229)

The partition of the submatrices and vectors into negligible localized mode expressions $|\mathbf{D}_i^{-1}| < \epsilon$ (where $\epsilon < 1$ is a small positive number) and accessible mode expressions leads to a significantly reduced system of equations (5), because there are only a few accessible modes in common waveguide structures. Moreover, the block matrices $\tilde{\mathbf{P}}_i$ and $\tilde{\mathbf{Q}}_i$ reduced in this way can be precomputed advantageously for all sections for a certain dimension range and (or) range of frequency points; the matrix elements are then interpolated by (multidimensional) cubic splines for the optimization process.

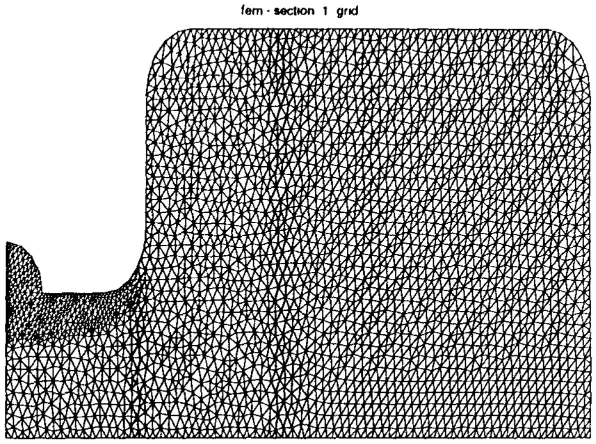


Fig. 4. FEM mesh of one quarter of the ridged waveguide low-pass filter structure in the ridged waveguide section. The additional gap of radius 0.3mm for the filter version of Fig 8 may demonstrate both the localized mesh refinement feature and the precision of the hybrid MM/FE method

III. Results

For the verification of the theory, a ridged waveguide filter (Fig. 2) with WR 229 in- and output ports has been optimized with the goal of 25 dB return loss between 3.4 and 4.2 GHz and more than 80 dB rejection (also for all higher-order modes) between 5.85 and 6.75 GHz. Localized higher-order modes up to the cut-off frequency of 100 GHz have been taken into account.

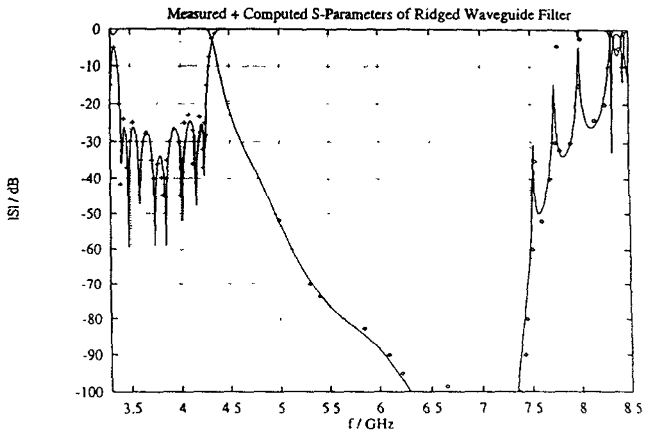


Fig. 5. S-parameters of an optimized ridged waveguide low-pass filter (WR-229). Theory and measurements.

The filter consists of alternate ridge and rectangular waveguide resonator sections. Fig. 4 illustrates the FEM mesh of one quarter of the FEM structure in the ridged waveguide section of the filter of Fig. 8.

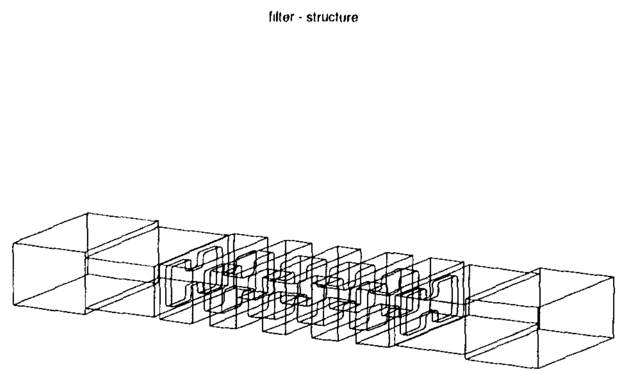


Fig. 6. Ridged waveguide low-pass filter (WR-75)

For the optimization, only five accessible mode in the homogeneous sections have been considered. About 10,000 iterations were necessary for the final results. The overall analysis (Fig. 5) has been checked by taking into account all accessible modes up to 50 GHz cut-off frequency in the homogeneous interim waveguide sections. The filter has been fabricated by computer controlled milling techniques. Excellent agreement with measurements may be demonstrated (Fig. 5).

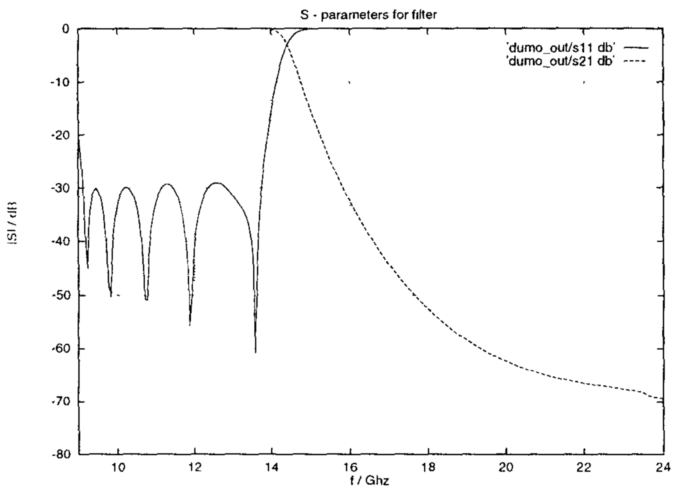


Fig. 7 S-parameters of the optimized ridged waveguide low-pass filter (WR-75)

A second example is a ridged waveguide filter (Fig. 6) with WR-75 waveguide in- and output ports. The filter has been optimized for more than 28dB return loss in the passband. The final results in Fig. 7 for the S-parameters have been checked with all higher-order modes up to 210 GHz cutoff frequency.

In order to demonstrate the high precision of the presented MMLU hybrid MM/FE CAD technique, an additional gap of only 0.3 mm radius has been assumed in the ridge section (Fig.4) of the WR-75 filter. The influence on the S-parameters can be clearly stated in Fig. 8.

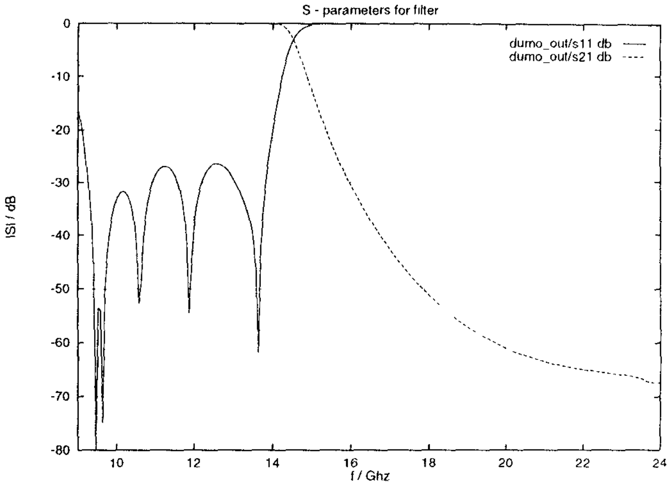


Fig. 8. Influence of a gap of 0.3mm radius (Fig. 4) on the S-parameters of the optimized ridged waveguide low-pass filter (WR-75)

IV. Conclusion

A new hybrid mode-matching/block-LU-decomposition (MMLU) FE method is introduced which achieves the very efficient CAD of the comprehensive class of waveguide components composed of step type discontinuities of arbitrary shape.

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